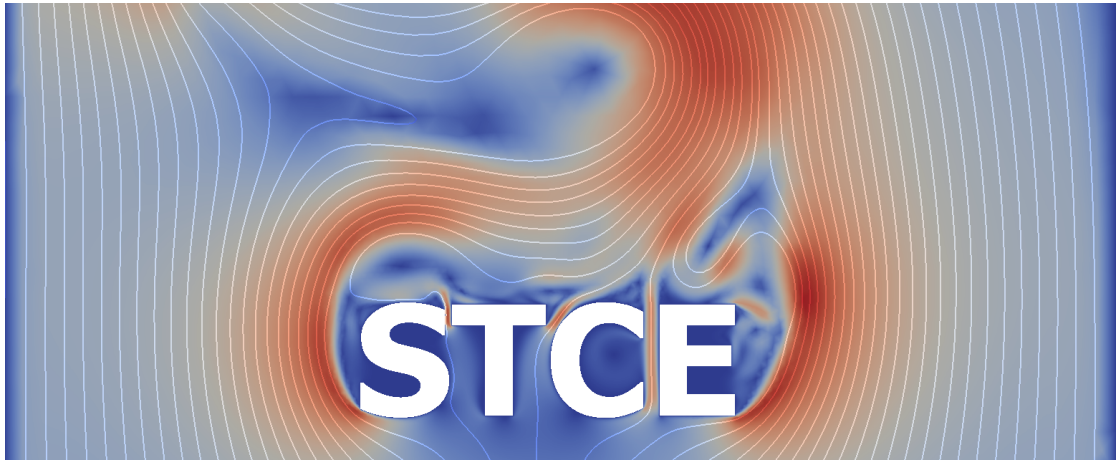

Significance Analysis for Numerical Models

... within the FET-open project SCoRPiO

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1st Workshop On Approximate Computing (WAPCO 2015), Amsterdam

Outline

SCoRPiO

Significance Analysis for Numerical Models

- Objective
- Algorithmic framework
- Significance Analysis in a nutshell
- Applications

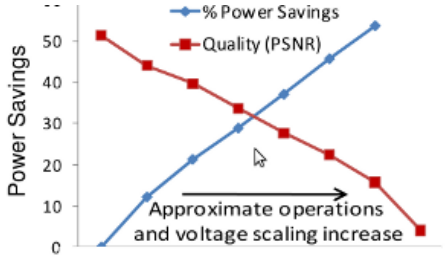
Outlook

SCoRPiO— Significance-Based Computing for Reliability and Power Optimization

SCoRPiO

Significance-Based Computing for
Reliability and Power Optimization

SEVENTH FRAMEWORK PROGRAMME



Power savings through controlled quality degradation, even in the presence of hardware faults

CERTH / IRETETH

RWTH AACHEN UNIVERSITY

Inria INVENTORS FOR THE DIGITAL WORLD

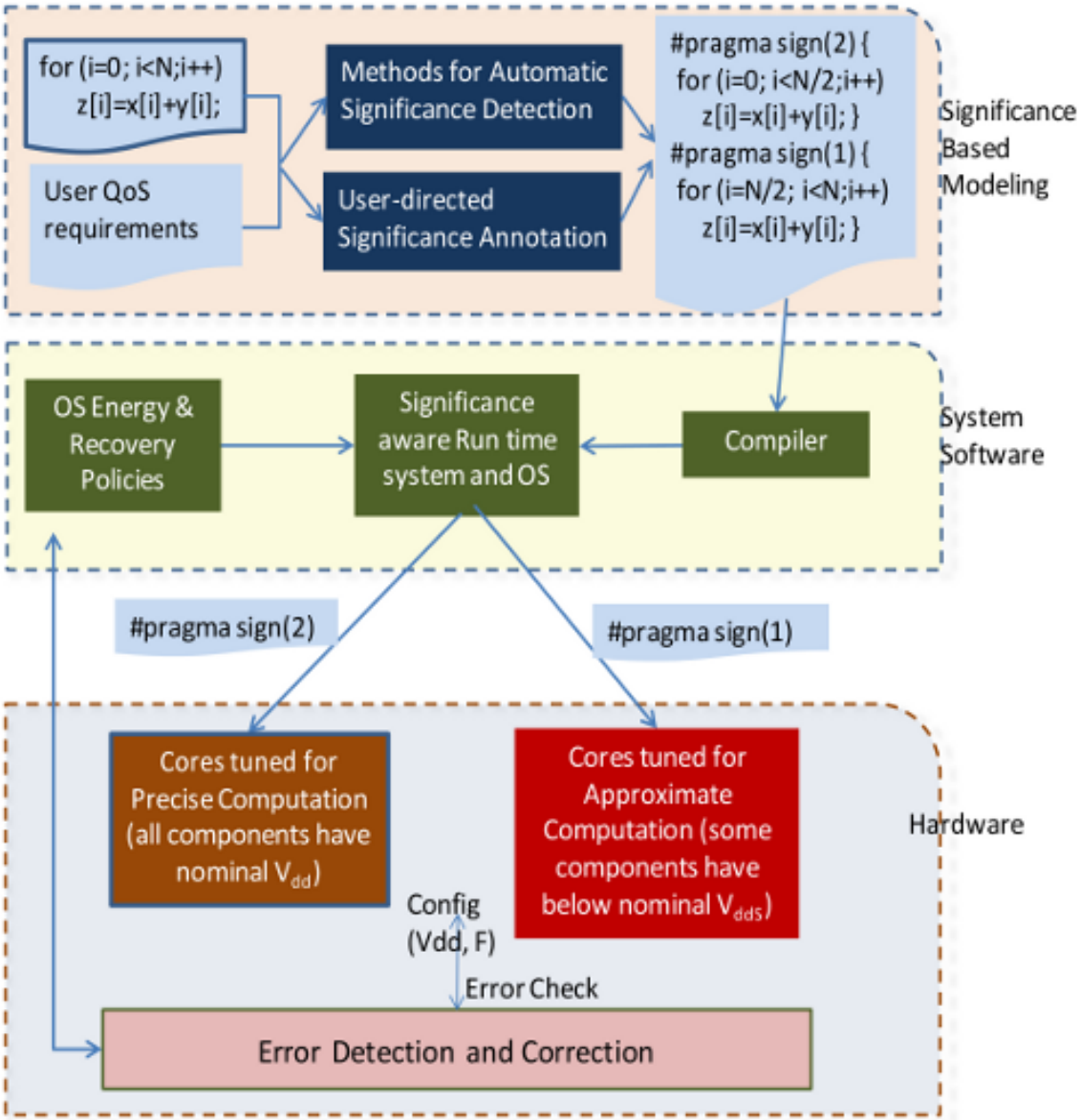
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Queen's University Belfast

imec

A multi-national, multi-disciplinary group of researchers

SCoRPiO
Project Structure



Significance Analysis of Numerical Models

Significance Analysis for Numerical Models
Objective

Given:

- Implementation of an algorithm (code) in C or C++
- Ranges of input values
- Rate of acceptable variability in output ranges (very application specific)

Objective:

Identify parts of the given code, that are **insignificant** wrt the user given allowed variability rate / bound / . . . for the output ranges.

A insignificant computation might be replaced by a cheaper computation

- using unreliable hardware
- using lower precision on slower, less power consuming hardware
- replacing the computations by a fixed (constant) representative
- reusing previously computed results for subsequent computations

(One) Motivation:

Iterative algorithms tend to find a good solution in early stages.

Significance Analysis for Numerical Models
Algorithmic framework

Key ingredients:

- **Interval arithmetic**
  - Computes guaranteed enclosures of true function values.
  - Value interval of a program variable describes the combined influence of input ranges on the variable (forward analysis).
  - Issues: overestimation (wrapping effect), relational operators.
- **Adjoint mode Algorithmic Differentiation**
  - An adjoint model can compute the individual influence of all inputs on a single output in one run (reverse analysis).
  - Issue: Differentiability is concept for continuous functions.

Technical setup:

- Exploit overloading of operators/intrinsic functions in C++
  - `dco/scorpio` (a specialization of `dco/c++` with `filib++`).
  - Run the executable with significance functionality.
- Nevertheless this a **compile time analysis** for the simulator in SCoRPiO
  - **Output** of significance analysis is **input** for the compiler of the simulator

Significance Analysis in a nutshell (I)

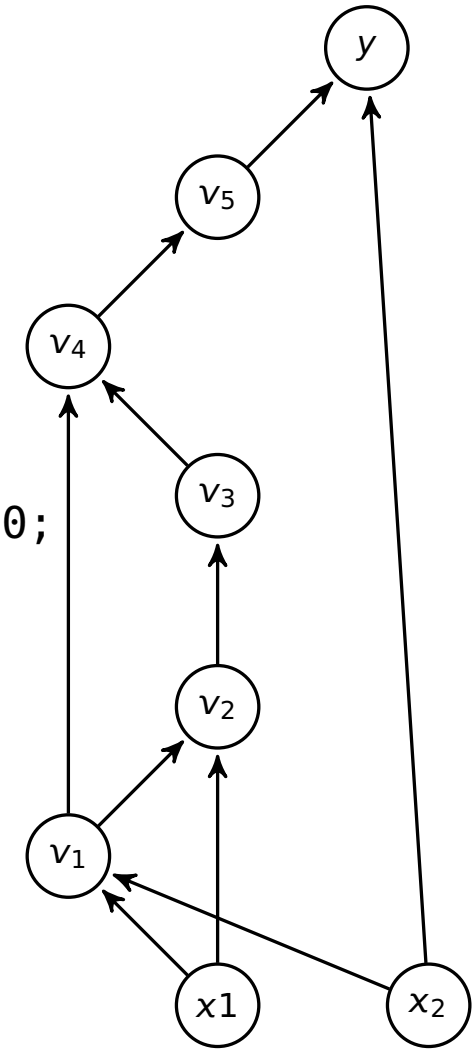
Input data:

$x1, x2 \in [1, 36000]$
$\epsilon = 2.5$

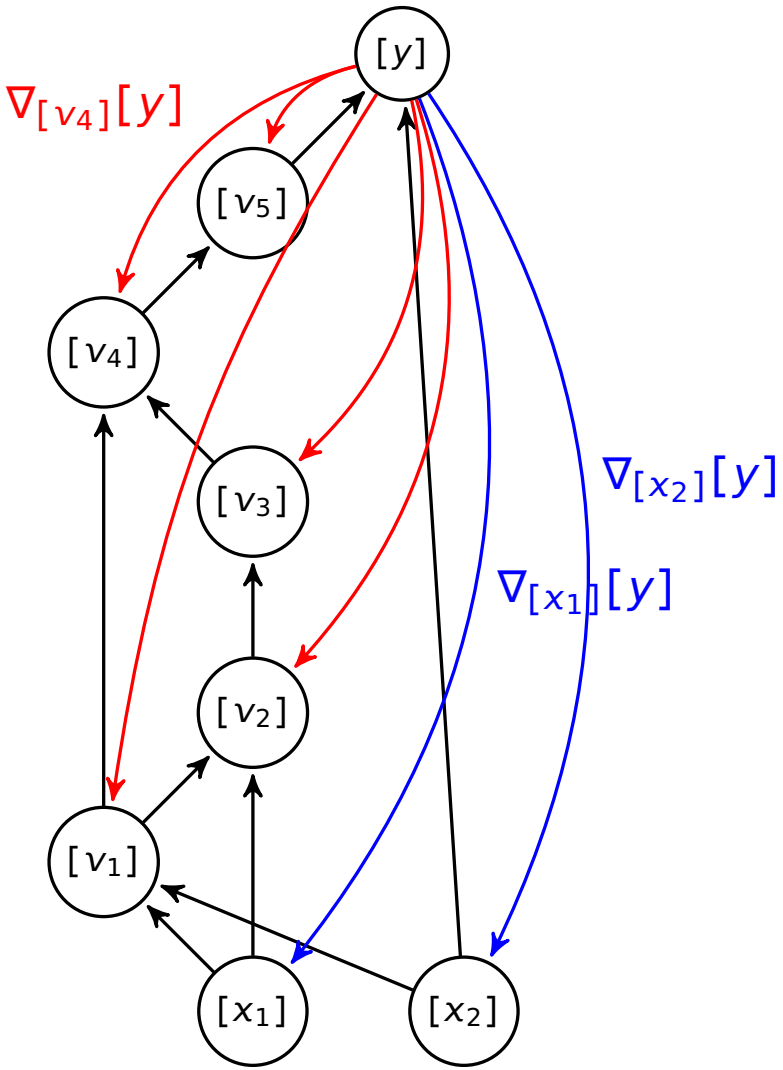
Original code:

```
v1 = x1 * x2;
v2 = sin(x1);
v3 = exp(v2);
v4 = v3 * v1;
v5 = log( v4 );
y  = v2/10 + x2/100;
```

Computational graph



dco/scorpio



Significance Analysis in a nutshell (II)

Input data:

$x1, x2 \in [1, 36000]$
$\epsilon = 2.5$

Original code:

```
v1 = x1 * x2;
v2 = sin(x1);
v3 = exp(v2);
v4 = v3 * v1;
v5 = log( v4 );
y  = v2/10 + x2/100;
```

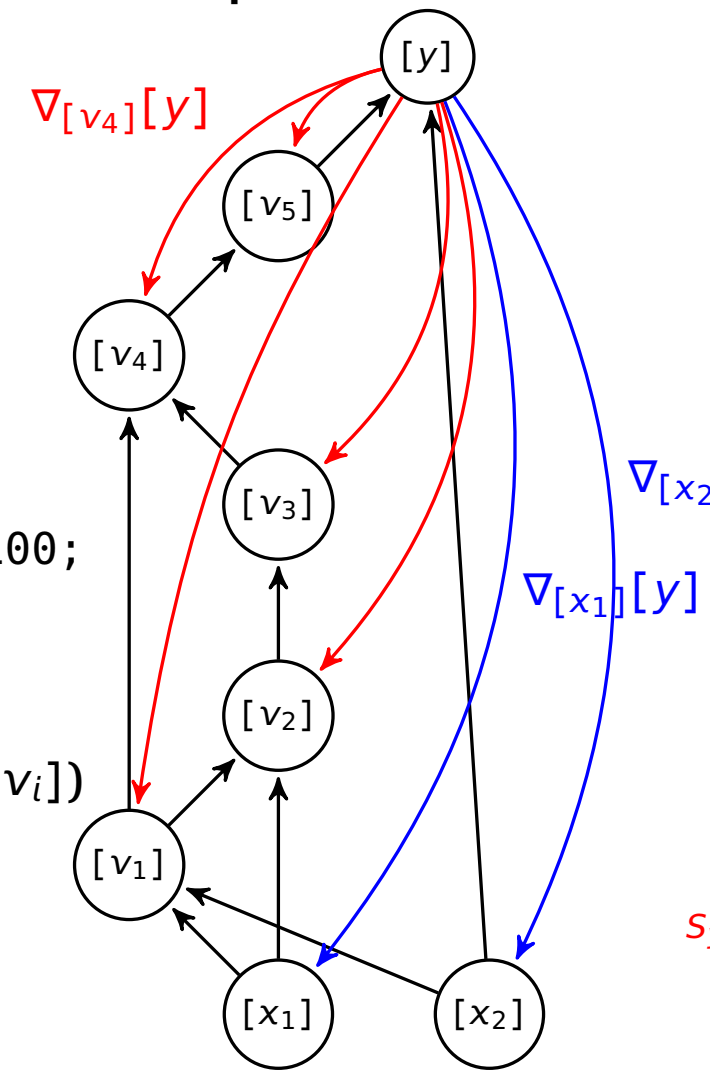
Significance:

$$S_y(v_i) = w([v_i] \cdot \nabla_{[y]}[v_i])$$

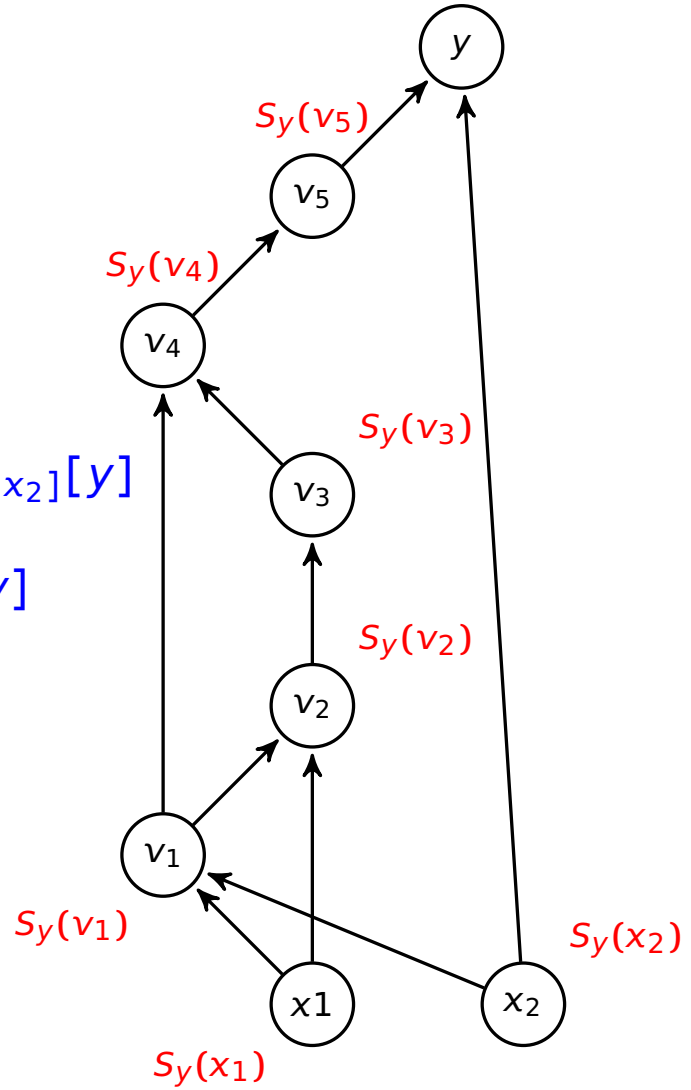
Insignificance test

$$S_y(v_i) \leq \epsilon$$

dco/scorpio



Significance



Significance Analysis in a nutshell (III)

Input data:

$x1, x2 \in [1, 36000]$
$\epsilon = 2.5$

Original code:

```
v1 = x1 * x2;
v2 = sin(x1);
v3 = exp(v2);
v4 = v3 * v1;
v5 = log( v4 );
y  = v2/10 + x2/100;
```

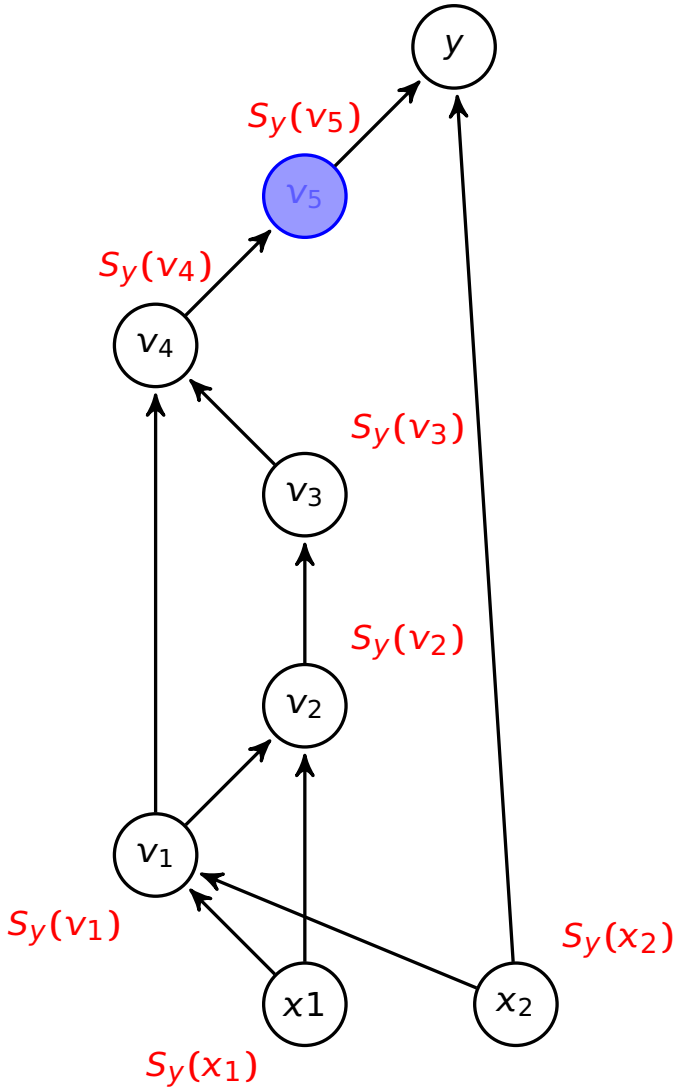
Significance:

$$S_y(v_i) = w([v_i] \cdot \nabla_{[y]}[v_i])$$

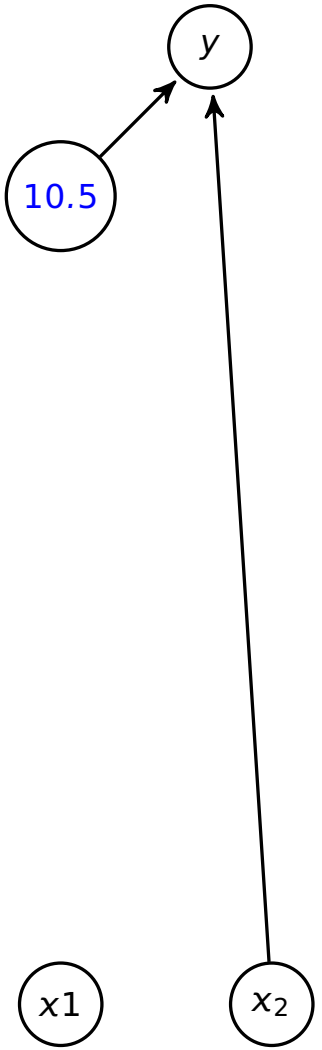
Insignificance test:

$$S_y(v_i) \leq \epsilon$$

Significance



Optimised DAG



Significance Analysis in a nutshell (IV)

Input data:

$x1, x2 \in [1, 36000]$
$\epsilon = 2.5$

Original code:

```
v1 = x1 * x2;
v2 = sin(x1);
v3 = exp(v2);
v4 = v3 * v1;
v5 = log( v4 );
y  = v2/10 + x2/100;
```

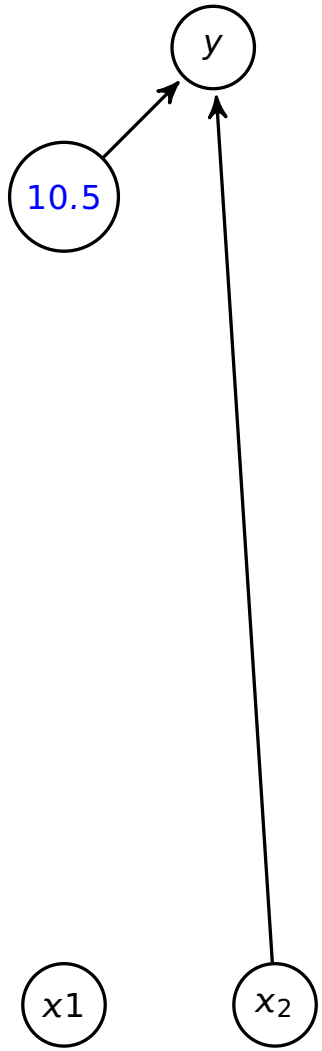
$\longrightarrow y \in [-0.09, 362]$

Approximating code:

```
v5 = 10.5;
y = v5/10 + x2/100;
```

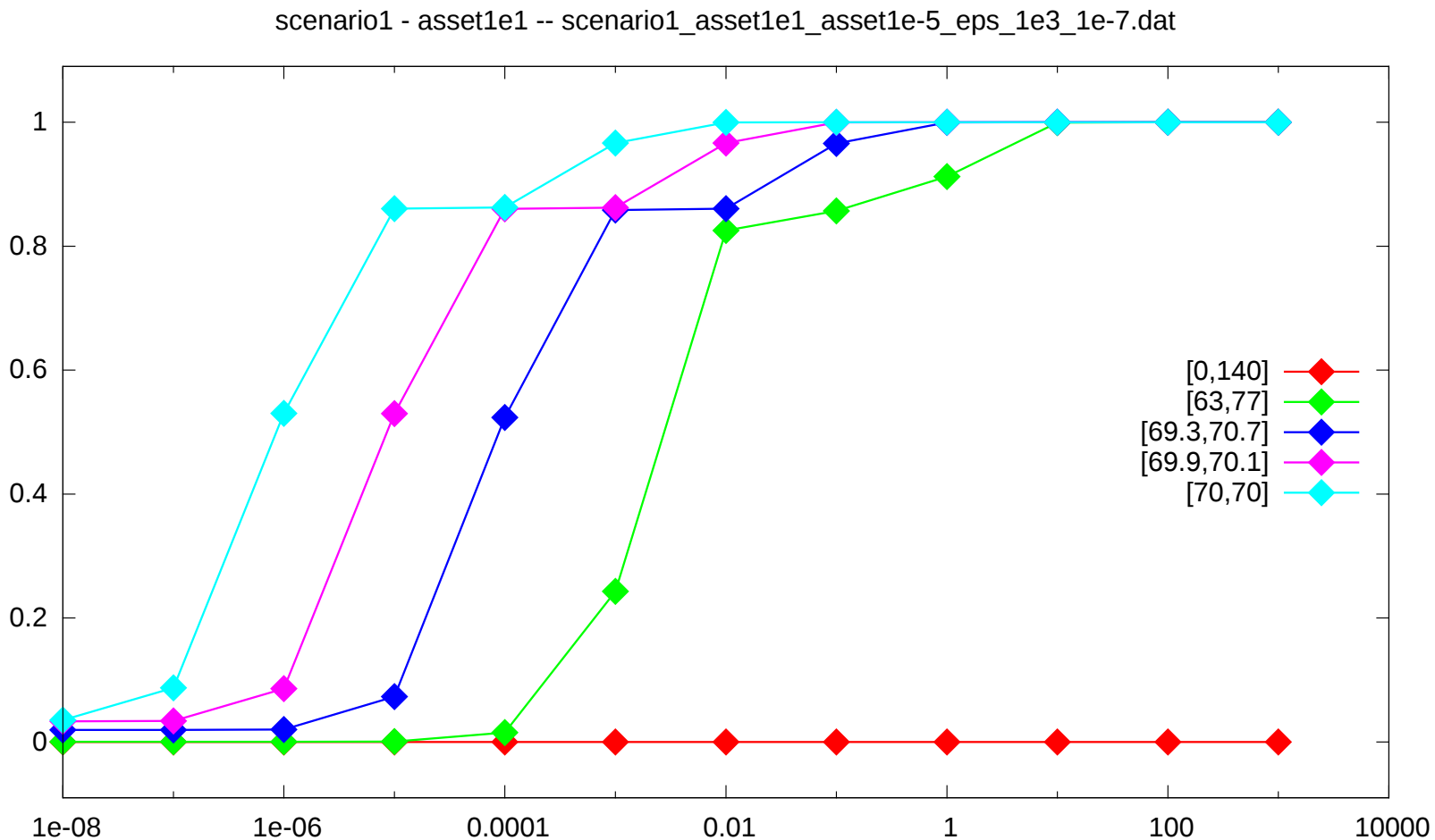
$\longrightarrow y \in [1.06, 361]$

Optimised DAG



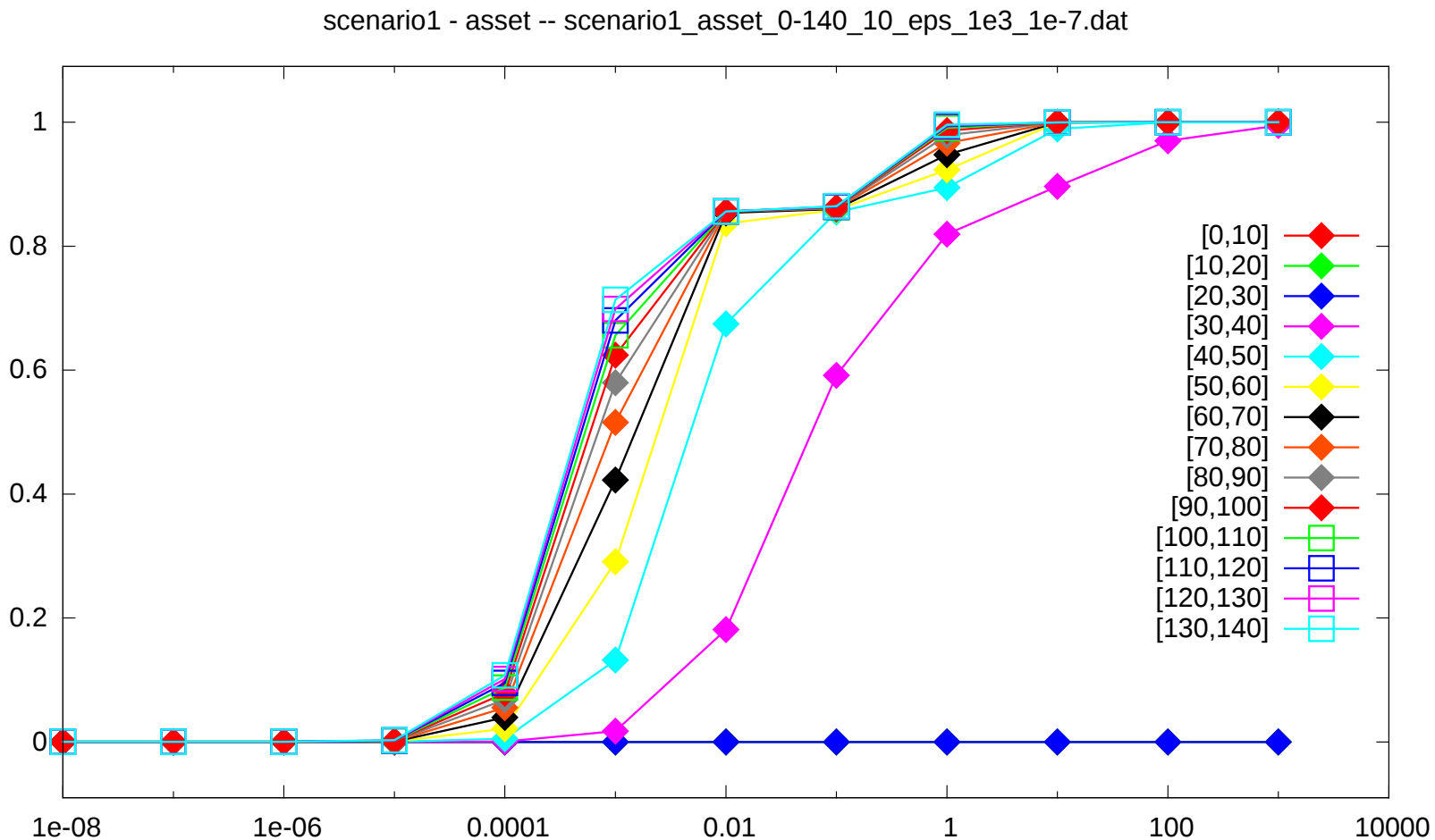
Hello Finance (Monte Carlo), Increasing input intervals

Percentage of insignificant program variables.



Hello Finance (Monte Carlo), Input interval cover

Percentage of insignificant program variables.



Hello Finance (PDE), in-house CFD flow solver

Interval - values and -adjoints even for point inputs : Complete $\boldsymbol{R}$

```
##################################################
        HelloFinance/PDE
##################################################

Price: [ ENTIRE ]
d_Price/d_K: [ ENTIRE ]
d_Price/d_S0: [ ENTIRE ]
d_Price/d_r: [ ENTIRE ]
```

We debugged that code to understand where the problem arises:

- No specific operation, constant growth
- At some point program variables became big and bigger . . .

  $\Longrightarrow$ wrapping effect (well known / feared)

Outlook

Outlook

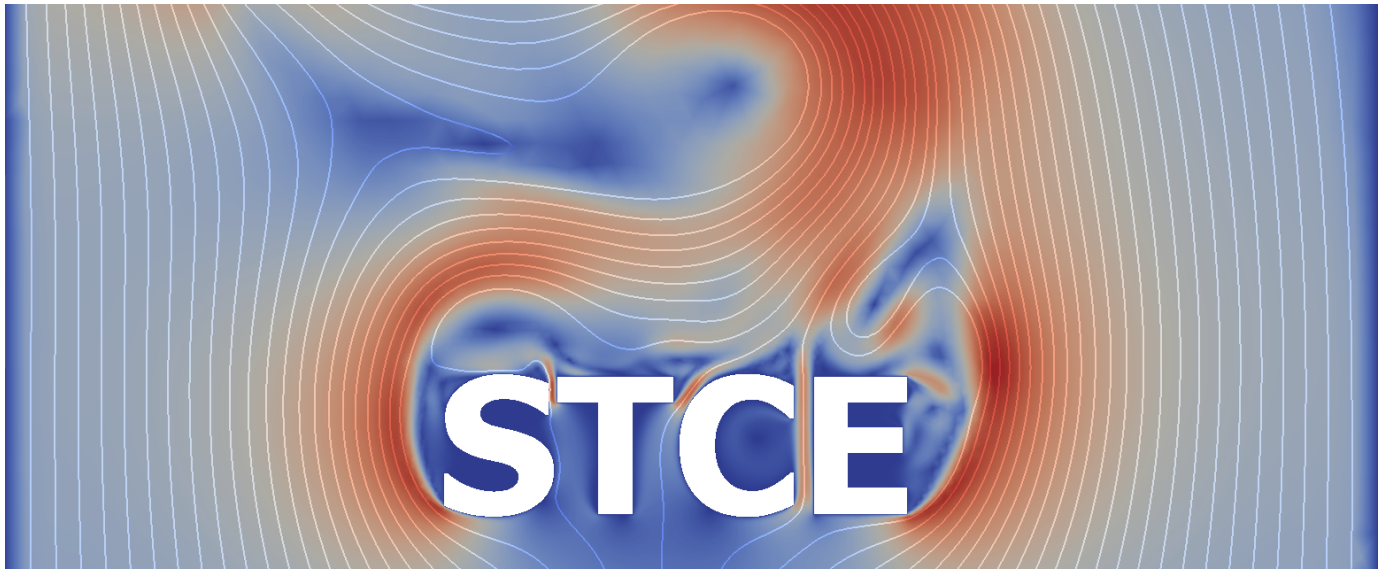
Goal: General purpose automatic significance analysis

We are working on an algorithm based on parallel recursive interval splitting of value intervals of program variables that will handle

- not feasible relational operators (ensure **control flow feasibility**),
- overestimation of value intervals (ensure **value feasibility**),
- overestimation of adjoint intervals (ensure **adjoint feasibility**).

Issues to address:

- More understanding of significance criteria is needed.
- New (scaled) significance criteria will be explored.
- How to rank feasible scenarios coming from the automatic significance analysis?
- What interactions exist between *control flow feasibility*, *value feasibility*, and *value feasibility*?
- How to split value intervals?
- How to schedule the parallel analysis processes efficiently?



Thank you very much!

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Resources

- **SCoRPiO deliverable D1.1** : Jan Riehme and Uwe Naumann. SCoRPiO Deliverable D1.1: Significance Based Computing Modeling. `www.scorpio-project.eu/wp-content/uploads/2014/07/Scorpio_D1.1.pdf`, June 2014.
- **`dco/c++`** : Software and Tools for Scientific Engineering, RWTH Aachen University, Germany. Derivative Code by Overloading in C++ (dco/c++). `http://www.stce.rwth-aachen.de/software/dco_cpp.html`.
- **`filib++`** : Michael Lerch, German Tischler, J"urgen Wolff von Gudenberg, Werner Hofschuster, and Walter Kr"amer. FILIB++ interval library. `www2.math.uni-wuppertal.de/~xsc/software/filib.html`.
- **Algorithmic Differentiation** :
  - A. Griewank and Andrea Walther. Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation. SIAM, 2nd edition, 2008.
  - U. Naumann. The Art of Differentiating Computer Programs. An Introduction to Algorithmic Differentiation. Software, Environments, and Tools. SIAM, 2011.
  - Community portal: `www.autodiff.org`
- **Interval Arithmetic** :
  - Ramon E. Moore. Interval Analysis. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1966.
  - Ramon E. Moore, R. Baker Kearfott, and Michael J. Cloud. Introduction to Interval Analysis. Society for Industrial and Applied Mathematics, 1 edition, 1 2009.